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Generalized Inverses for Finite-Horizon Tracking

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Generalized Inverses for Finite-horizon Tracking[1]

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Abstract

Control and communication issues are traditionally "decoupled" in discussions of decision and control problems, as this simplifies the analysis and generally works well for classical models. This fundamental assumption deserves re-examination as control applications spread into new areas where system complexity is significant. Such areas include the coordinated control of aerial vehicles (UAV's), MEMS devices, multi-joint manipulators and other settings where many systems must share the attention of a decision-maker. We consider a new class of sampled-data systems (termed "computer-controlled systems") that offer the possibility of jointly optimizing between control and communication goals. Computer-controlled LTI systems can be viewed as linear operators between appropriate inner-product spaces. The generalized inverses of these operators are used to solve a class of finite-horizon tracking problems.

1 Introduction

With the increased adoption of digital computers as tools for automatic control, sampled-data systems have become ubiquitous. Such systems typically include a digital controller interfaced to a continuous-time physical plant. The use of a digital controller limits controller-plant communication in the sense that communication only occurs at discrete times. An additional constraint emerges if the communication bus that is available to the controller has fewer channels than the number of inputs of the plant. In that case, the controller must choose which inputs to update at a particular time. In practice, one usually ensures that the controller generates commands with a "sufficiently high" frequency (in a Nyquist sense) so that the effects of communication constraints on the control problem become less pronounced. However, this "decoupling" of the control and communication problems must be re-examined as we seek to understand control problems such as the coordination of swarms of vehicles, MEMS arrays and other systems in which inputs/outputs must share the attention of a decision-maker. It is exactly this "sharing of attention" that must be addressed if such systems are to be used efficiently and effectively.

We investigate a class of sampled-data systems with communication constraints for which control and communication are intrinsically coupled. For such systems, we consider the problem of tracking a-priori known, finite-horizon outputs. This is essentially a feed-forward control problem, also known as "preview tracking" (see [16], [15], [9] and references therein). Our approach is novel in that it focuses on achieving optimal tracking performance with no assumptions on the controller's bandwidth and on bringing forth the *explicit dependence* of the optimal control on controller-plant communication. Using an operator-theoretic approach to computer-controlled systems (see also [12]), we pose the problem of finite-horizon tracking as a least-squares matching problem and obtain the solution by constructing a class of generalized inverses for computer-controlled systems. These ideas will be made precise in Sec. 2. Previous work on systems with communication constraints can be found in [1], [17], [3] and [11]. The issues of distributed computation, control and estimation with limited bandwidth are addressed in [4], [18]. For modeling and analysis of sampled-data systems, see [2], [8], [19] and [12]. In the context of control systems, models for "attention" were introduced in [5] and [6]. For some of the early work on the use of generalized inverses in systems theory see [14], [20].

2 A Prototype Computer-Controlled System

In this section, we propose a model for computer-controlled systems based on the idea of an "attention sequence" (originally introduced in [5]) which is used to direct communications between controller and system. With respect to notation, we use $\ell^k(N)$ to denote the space of finite sequences of vectors in $\mathbb{R}^k$ with

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$u = \{u(1), u(2), ..., u(N)\}$ being a typical element of $\ell^k(N)$. Elements of individual vectors in a sequence are denoted by subscripts (e.g. $u(2)_1$). The space of square-integrable $\mathbb{R}^p$-valued continuous signals on $[0,T]$ is denoted by $L_2^p[0,T]$. Norms in these two spaces are those induced by the usual inner products:
$\langle u, v\rangle_{\ell^m(N)} = \sum_{k=0}^{N-1} u^T(k)v(k)$ and
$\langle y, z\rangle_{L_2^p[0,T]} = \int_0^T y^T(t)z(t)dt$
for $u, v \in \ell^m(N)$ and $y, z \in L_2^p[0,T]$. By $Rat^{m\times p}(n)$ we understand the space of $m \times p$ matrices whose elements are proper rational functions with denominators of degree n.

Consider a continuous-time LTI system that is driven by a computer or other digital controller (Fig. 1).
• The controller cannot provide continuous inputs to the LTI system; instead, commands are sent to the system every Δ time units, via a zero-order-hold stage.
• The dimension of the communication bus that carries controller-generated inputs may be less than the input dimension of the LTI plant. As a result, the controller must choose which of the input signals to update at every step.

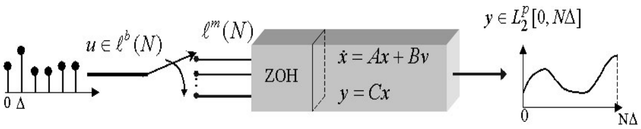


Figure 1: A computer-controlled LTI system

We will use b to denote the "size" of the communication bus with $u \in \ell^b(N)$ being a controller-generated sequence on that bus. We will ignore quantization effects associated with the communication bus.

Definition 1 *A* **computer-controlled LTI system** *is a triple $(G(s), \Delta, b) \in Rat^{p\times m}(n) \times \mathbb{R}_+ \times \mathbb{N}_m^*$ where:*
• *$G(s)$ is the transfer function of an n^{th}-order LTI system with input $v(t) \in \mathbb{R}^m$ and output $y(t) \in \mathbb{R}^p$. The LTI system is driven by a digital controller through a zero-order-hold stage.*
• *$b \le m$ is the dimension of the communication bus connecting the controller to the zero-order hold.*
• *$\frac{1}{\Delta}$ is the controller rate.*

In this work we will take the underlying system to be LTI. However, computer-controlled systems need not always be linear and one could amend Def. 1 (and Fig. 1) by replacing $G(s)$ by a non-linear system. We will use the terms "narrow" and "wide" to describe the communication bus when $b < m$ and $b = m$ respectively. When the communication bus is narrow, one possibility is to choose a sequence of operations for the switch (see Fig. 1) that selects which system inputs are to be updated at a particular time.

Definition 2 *By an* **attention sequence** *of length N and width m, we understand an element of*

$$\mathbb{E}^{m\times N} = \{(\sigma(0), \sigma(1), ..., \sigma(N-1)) : \sigma(i) \in \{0,1\}^m\}$$

i.e. an ordered set of N elements of $\{0,1\}^m$.

In the context of a computer-controlled system, the vectors $\sigma(i)$ of an attention sequence σ are to be interpreted as indicating which elements of the system input $v(t)$ are to be updated by the controller at $t = i\Delta$, $i = 0, \dots, N-1$. The attention sequence and switch essentially implement a de-multiplexer.

Definition 3 *Consider a computer-controlled LTI system with $b, m \in \mathbb{N}$ being the dimensions of the communication bus and system input respectively ($b \le m$). An attention sequence $\sigma \in \mathbb{E}^{m\times N}$ is* **admissible** *if:*
• *At least one but no more than b of the system inputs are updated by the controller at every step:*
$0 < \|\sigma(i)\|^2 \le b \quad \forall i = 0, ..., N-1$
• *The controller communicates with all inputs of the linear system: $Span\{\sigma(0), \dots \sigma(N-1)\} = \mathbb{R}^m$*

It should be noted that computer-controlled LTI systems are time-varying because they incorporate a zero-order-hold stage *and* because controller-plant communication is time-dependent.

3 An N-step Look-ahead Tracking Problem

Armed with the definitions of Sec. 2 we can now formulate the following output tracking problem:

Problem Statement 1 *Given a computer-controlled LTI system $(G(s), \Delta, b)$, an integer $N > 0$, a desired output $y_d \in L_2^p[0, N\Delta]$, and an admissible attention sequence $\sigma \in \mathbb{E}^{m\times N}$, find the input sequence $u \in \ell^m(N)$ that minimizes $\|y_d - y\|$.*

Before we present the solution to the above problem, recall that an LTI system preceded by a zero-order hold, can be regarded as a linear, time-varying operator that maps input sequences to outputs:

Definition 4 *Given an LTI system $G(s) \in Rat^{p\times m}(n)$, an input-sampling period Δ, and an integer $N > 0$, we define the* **input-output map** *of $(G(s), \Delta, N)$ to be $\Lambda_{(G,\Delta,N)}(t) : \ell^m(N) \longrightarrow L_2^p[0, N\Delta]$ with*

$$y(t) = \Lambda_{(G(s),\Delta,N)}(t)u = \sum_{k=0}^{N-1} \phi_\Delta(t - k\Delta)u(k) \qquad (1)$$

for $u \in \ell^m(N)$, $y \in L_2^p[0, N\Delta]$, where

$$\phi_\Delta(t) = \begin{cases} \int_0^{min(t,\Delta)} Ce^{A(t-\tau)}Bd\tau & t \ge 0 \\ 0 & t < 0 \end{cases} \qquad (2)$$

and the triple $(B, A, C) \in \mathbb{R}^{m\times n} \times \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times p}$ is a state-space realization of $G(s)$.

A similar formulation (applied to sampled-data systems) is found in [12]. The function ϕ_Δ is the response of the $G(s)$ to a unit pulse of duration Δ. Without loss of generality, we assume $x(0) = 0$ for the state of the LTI system. We note that the range space of $\Lambda_{(G,\Delta,N)}$ is infinite-dimensional. It is clear from Eq. 1 that the range of Λ contains only those elements in $L_2^p[0, N\Delta]$ that are linear combinations of ϕ_Δ and its Δ-translates, therefore Λ is not surjective. Having chosen inner products in $\ell^m(N)$ and $L_2^p[0,T]$, the following can be easily verified:

Observation 1 *The adjoint operator of Λ (Def. 4) is*

$$\begin{aligned} \Lambda^*_{(G,\Delta,N)} &: L_2^p[0,N\Delta] \longrightarrow \ell^m(N) \\ (\Lambda^* y)(j) &= \int_0^{N\Delta} \phi_\Delta^T(t - j\Delta) y(t) dt \end{aligned} \tag{3}$$

for $0 \le j \le N-1, \quad y \in L_2^p[0, N\Delta]$.

In the following, we will sometimes abuse the notation by writing Λ (Λ^*) as an abbreviation for $\Lambda_{(G(s),\Delta,N)}(t)$ ($\Lambda^*_{(G(s),\Delta,N)}(t)$); we always have in mind that Λ, Λ^* are defined for a particular choice of $G(s), \Delta$ and N. We now focus on the controller-plant communication.

Lemma 1 *Consider a computer-controlled LTI system and let $b, m \in \mathbb{N}$ be the dimensions of the communication bus and system input respectively, with $b \le m$. An admissible attention sequence $\sigma \in \mathbb{E}^{m\times N}$ together with an integer $N > 0$ define a 1-to-1* **attention map**

$$D_{(b,m,N)}(\sigma) : \ell^b(N) \longrightarrow \ell^m(N) \tag{4}$$

such that for $u \in \ell^b(N)$ and $0 < k < N$:

$$D_{(b,m,N)}(\sigma)u(k)_i \ne D_{(b,m,N)}(\sigma)u(k-1)_i \tag{5}$$

for at most b indices $i \in 1, ..., m$. If we identify elements in $\ell^m(N)$ with vectors in $\mathbb{R}^{m\cdot N}$ then

$$D_{(b,m,N)}(\sigma) = \tilde{D}(\sigma) \in \mathbb{R}^{Nm\times Nb} \tag{6}$$

$$\tilde{D}_{ij}(\sigma) = \begin{cases} 1 \textit{ if } & \lfloor \frac{i-1}{m} \rfloor \ge \lfloor \frac{j-1}{b} \rfloor \ , \\ & \sigma(\lfloor \frac{j-1}{b} \rfloor)_{\beta(i,m)} = 1 \\ & \textit{and } \sum_{q=1}^{\beta(i,m)} \sigma(\lfloor \frac{j-1}{b} \rfloor)_q = \beta(j,b) \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$

for $1 \le i \le Nm, 1 \le j \le Nb$, and $\beta(i,j) \triangleq i - \lfloor \frac{i}{j+1} \rfloor j$ for i, j integers.

Proof: *Let $u \in \ell^b(N)$ be a controller-generated input sequence. The elements of $u(i)$ will be used to update the inputs of the zero-order-hold stage according to σ so that $u' \in \ell^m(N)$ will be the corresponding sequence appearing at the zero-order-hold stage, after demultiplexing. Because of the communication constraint $b < m$, the controller can only update b of the m elements of $u'(k)$ for $k = 0, ..., N-1$, according to σ. Therefore, the only possible input sequences u' satisfy:*

$$u'(k)_i \ne u'(k-1)_i \tag{8}$$

for at most b indices $i \in 1, .., m$.

For the elements of the attention sequence, we can write $\sigma(k) = \sum_{j=1}^{b_k} e_{k_j}$ where e_i is the standard basis in $\mathbb{R}^m$, $b_k = \sum_i \sigma(k)_i$ and $1 \le k_j \le m$ such that $\sigma(k)_j = 1$. Then,

$$u'(k) = \sum_{j=1}^{b_k} e_{k_j} u_j(k) = E(k)u(k) \tag{9}$$

where $E(k) \triangleq \left[\begin{array}{c|c|c|c} e_{k_1} & e_{k_2} & \dots & e_{k_{b(k)}} \end{array} \right] \in \{0,1\}^{m\times b}$. It is now clear that u and u' are related by a linear map and that all elements of u' except $u'(0)$ are determined by u together with the binary matrices $E(k)$. This relation can be expressed as $u' = D_{(b,m,N)}(\sigma)u$ where without loss of generality we have assumed that of $u'(0)_i = 0$ for i such that $\sigma(0)_i = 0$. The admissibility of σ guarantees that at least one but no more than b elements of $u'(k)$ will be updated for every $k = 0, ..., N-1$. Equivalently, the matrices $E(k)$ have maximum column-rank. Therefore, for $u, v \in \ell^b(N)$, $u \ne v \Rightarrow Du \ne Dv$ and D is 1-to-1. As with Λ, we have used D to abbreviate $D_{b,m,N}(\sigma)$. Identify $\ell^b(N)$ with $\mathbb{R}^{b\cdot N}$. Then $u = [u^T(1), ..., u^T(N)]^T$. The matrix $\tilde{D}(\sigma)$ which realizes D, can now be constructed as:

$$\tilde{D}(\sigma) = \left[\begin{array}{c|c|c|c} \alpha(1) & \alpha(2) & \cdots & \alpha(N) \end{array} \right] \tag{10}$$

where $\alpha(i) = E(i) \otimes \sum_i^N e_j$, e_j are the standard basis vectors in $\mathbb{R}^N$ and $\otimes$ denotes the Kronecker product. Equation 7 follows from $\tilde{D}(\sigma)$. ■

We note that $\tilde{D}$ as described in Eq. 10 leaves undetermined $m - b$ elements of the initial input $u'(0) \in \mathbb{R}^m$. Those elements of $u'(0)$ can be taken to have fixed initial values. If the onset of a control task can be delayed until all initial inputs have been communicated then we can modify the attention sequence by setting $\sigma(0)_i = 1$ for all $0 \le i \le m$. The following result gives the solution to the N-step look-ahead tracking problem for an LTI computer-controlled system by constructing a generalized inverse for that system.

Theorem 1 *Let $(G(s), \Delta, b)$ be a computer-controlled LTI system with $G(s) \in Rat^{p\times m}(n)$, $m \le p$ and $rank(G(s)) = m$. Then the solution to the N-step look-ahead tracking problem is:*

$$u_* = (D^T \Lambda^* \Lambda D)^{-1} D^T \Lambda^* (y_d - y_{ic}) \tag{11}$$

where $\Lambda(t)$ is the input-output map of $(G(s), \Delta, N)$, $D(\sigma)$ is the attention map defined by the sequence σ and y_{ic} is the effect of the initial conditions of $G(s)$.

Proof: *At first, assume zero initial conditions for the state $x(0)$ of the LTI system $G(s)$. Input sequences generated by the controller are mapped to outputs of $(G(s), \Delta, b)$ by*

$$\begin{aligned} (\Lambda D) &: \ell^b(N) \longrightarrow L_2^p[0, N\Delta] \\ y(t) &= \Lambda(t) D u \end{aligned} \tag{12}$$

For σ admissible, D is 1-to-1. It is enough to show that Λ is 1-to-1 as well. Then, ΛD will be 1-to-1, $D^T\Lambda^*\Lambda D$ will be invertible and u_* can be obtained as the least-squares solution of the equation $\Lambda Du = y_d$. To show Λ is 1-to-1, it's enough to show that the scalar $\gamma = \left\|\sum_{k=0}^{N-1}\phi_\Delta(t-k\Delta)u(k)\right\| = \|\Lambda u\|$ is strictly positive for $\|u\| \neq 0$. Then $\gamma^2 = \langle u, \Lambda^*\Lambda u\rangle_{\ell^m(N)} > 0$ for all nonzero u and therefore Λ is kernel-free. We note that the function $\phi_\Delta(t)$ and its Δ-translates are a basis for the range space of Λ and independent, in the sense that none of them can be expressed as a linear combination of the others. To show $\gamma > 0$, choose any nonzero $u \in \ell^m(N)$. Let $0 \leq j \leq N-1$ be the smallest index such that $u(k) = 0$ for $0 \leq k < j$, or $j = 0$ of no such index exists. Then, $\gamma \geq \|\phi_\Delta(t-j\Delta)u(j)\|_{L_2^p[0,\Delta]}$ because $\phi_\Delta(t-j\Delta)$ is outside the span of the other translates of $\phi_\Delta(t)$. If the transfer function $G(s)$ has rank m, the non-zero input $u(j)$ will produce a non-zero output so $\|\phi_\Delta(t-j\Delta)u(j)\|_{L_2^p[0,\Delta]} > 0$ and $\gamma > 0$. We conclude that ΛD is 1-to-1 and has a generalized inverse given by

$$(\Lambda D)^\# = (D^T\Lambda^*\Lambda D)^{-1}D^T\Lambda^* \tag{13}$$

If $x(0) \neq 0$, matters must be modified by tracking $y_d - y_{ic}$ instead of y_d, where

$$y_{ic}(t) \triangleq Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}B\tilde{u}dt, \quad t \in [0, N\Delta] \tag{14}$$

and $\tilde{u} \in \mathbb{R}^m$ is the initial conditions for the $m-b$ elements of $u(0)$ that are not updated at $t = 0$. ■

Equation 13 can be considered an analogue of the well-known formula for the left pseudo-inverse of an operator $M : \mathbb{R}^n \to \mathbb{R}^m$, with $m > n$, $rank(M) = n$:

$$M^\# = (M^TM)^{-1}M^T$$

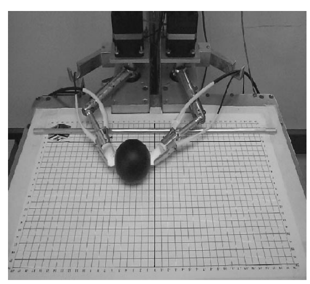
In practice $G(s)$ should be stable or stabilized by feedback. Although we do not address feedback here, results on the stabilization of computer-controlled systems can be found in [5], [11] and [10]. We note that the solution to the N-step look-ahead problem depends on the choice of attention sequence, corresponding to the fact that control and communication are intrinsically coupled in computer-controlled systems. Choosing the attention sequence now offers the possibility of jointly optimizing between control and communication:

Problem Statement 2 *Given a computer-controlled LTI system $(G(s), \Delta, b)$, a desired output $y_d \in L_2^p[0, N\Delta]$, and $N \in \mathbb{N}^*$, find the input $u \in \ell^m(N)$ and the attention sequence $\sigma \in \mathbb{E}^{N\times m}$ that minimize $\|y_d - y\|_{L_2^p[0,N\Delta]}$.*

The number of possible attention sequences is finite and therefore the minimum exists, although it may not be unique. This problem has not been solved and – except for trivial cases – cannot be "split" into separate sub-problems, one involving optimal control, the other optimal communication. Changes in the attention sequence result in changes in the optimal input sequence.

4 A Motion Control System with Limited Communication

Figure 2-a shows the Harvard Robotics Lab planar manipulator. The manipulator consists of two robotic fingers, each having two joints. The joints are driven by motors that contain integrated PID controllers, operating at $4KHz$. A computer communicates with the motors through an RS-232 serial port. All four motors are connected to the same serial port so that the computer can address *one motor at a time*, at a rate of $20Hz$. Possible motor commands include position and velocity setpoints, sensing of position or velocity as well as setting coefficient values for the local PID controller. Deformable tactile sensors are attached to the fingertips [7]. The sensors can localize contact with an accuracy of $1.5mm$ and can provide a rough estimate of local curvature at a contact at a rate of $10Hz$. An overhead camera tracks objects on a table, at a rate of $30Hz$. The position of an object can be determined within $3mm$, limited by the resolution of the overhead camera.



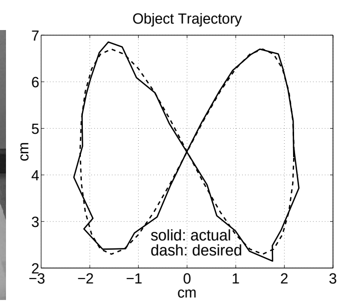


Figure 2: (a): Planar manipulator, (b) Kinematic exploration

The manipulator uses joint, visual and tactile feedback to locate, grasp and move objects along user-specified trajectories [10]. For the experiments described here, we used a $50gr$ spherical object and required that it follow a "figure-8" path:

$$\begin{aligned} x_d(t) &= 4.2\cos(t) \\ y_d(t) &= 4.5 + 1.7\sin(2t), \quad t \in [0,1]sec \end{aligned} \tag{15}$$

with x_d, y_d measured in *cm*. Using the kinematics of the manipulator (see for example [13], [10] and references therein), the trajectory was first sampled using a sequence of forty uniformly spaced setpoints and the manipulator moved the object through each setpoint. Figure 2-b shows the trajectory that was traced by the geometric center of the object. The dashed and solid

curves represent the desired and actual paths respectively. This kinematic exploration of the desired trajectory produced a set of desired joint position and velocity signals. Those signals were used together with a linear model of each joint, to arrive at a set of actuator commands (shown in Fig. 3-a) which would produce the desired joint velocities and ultimately the trajectory of Eq. 15. The four inputs were labeled as follows: 1-left proximal, 2-left distal, 3-right proximal, 4-right distal. We used the local PID controllers (embedded in each actuator) to implement a feedback linearization scheme, modeling each joint as a linear system. Coupling effects among joints were ignored.

4.1 Dynamic Performance

Figure 2-b shows good tracking performance in a geometric sense, meaning that the object came very close to the desired locus of points but it did so moving slowly (approximately $10sec$ to complete the figure-8). If we require that the trajectory of Eq. 15 be followed in real time, then the inputs of Fig. 3-a must be applied to the motors. Of course, those inputs are not feasible because they require communication rates higher than the available $20Hz$ (5 commands/sec per actuator, on average). We now need a method for computing the input sequence that results in minimum deviation from the desired trajectory. Depending on the required motions of the fingers, some joints may require more frequent communication than others. Therefore, we do not expect uniform sampling of all actuators to be an optimal strategy. We present the results obtained using two different attention sequences. Theorem 1 was used to compute the optimal input velocities for each attention sequence ($\Delta = 0.05$, $N = 20$). The optimal inputs were applied to the motors and the resulting object trajectory was recorded and compared with the desired one. We computed the tracking error as the magnitude of the total area between the desired and actual trajectories. In addition, the "joint tracking error" was computed as the L_2 error between the desired and actual joint trajectories.

4.2 Uniform Attention

We selected the attention sequence

$$\sigma = (e_1, e_2, e_3, e_4, e_1, \ldots, e_4) \tag{16}$$

using basis vectors to indicate which actuator is updated at each step. To obtain a basis for comparison, we first computed an input sequence by "averaging" each ideal input signal (Fig. 3-b) over the time intervals between consecutive updates of that input. Fig. 3-b shows the object trajectory achieved using those averages as inputs. The tracking error was $12.1cm^2$ while the joint tracking error was 0.32.

Next, the optimal inputs for the figure-8 trajectory were computed and transmitted to the motors. In this case (Fig. 4-a), the area tracking error was $5.48cm^2$,

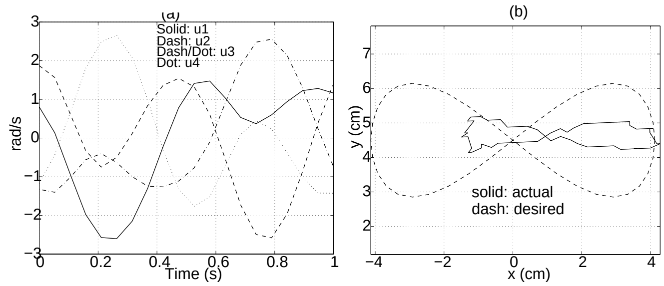


Figure 3: (a) Ideal (continuous-time) inputs (b) Object trajectory using "averaging"

an improvement by a factor of 2 over "averaging". The joint tracking error was 0.16. We observe that the object's actual trajectory was not a closed curve. This is because our least-squares formulation of the trajectory tracking problem did not include constraints on the final conditions of the control system.

4.3 Non-uniform Attention

The figure-8-tracking experiment was performed again, this time using an attention sequence that devotes $10\%, 35\%, 15\%$ and 40% to inputs $1, 2, 3$ and 4 respectively:

$$\begin{aligned}\sigma \;=\; & (e_3, e_4, e_1, e_2, e_4, e_4, e_2, e_4, e_1, e_2, e_4, e_2, \\ & e_3, e_4, e_2, e_4, e_2, e_3, e_4, e_2)\end{aligned} \tag{17}$$

Notice that distal joints (inputs 2 and 4) are updated more frequently than proximal joints. We arrived at this choice of communication sequence by observing the ideal (but infeasible) actuator inputs (in Fig. 3-a). For each time interval of length $\Delta = 0.05sec$, we allocated communication cycles using as a guide the amount of rotation required by each joint over that interval.

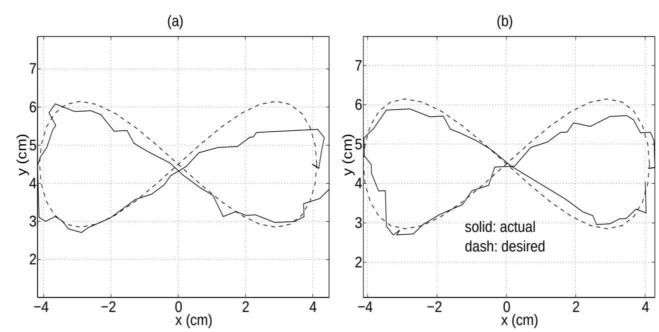


Figure 4: Object trajectory with (a) Uniform and (b) Non-uniform attention

When averaging was used, tracking performance was similar to that obtained with uniform attention. When the optimal inputs were used, tracking performance was slightly improved over what was achieved with uniform attention. The area tracking error was $3.15cm^2$ while the joint tracking error dropped to 0.06. The corresponding object trajectory is shown in Fig. 4-b.

We were unable to find an attention sequence that significantly improved over uniform attention. Most likely, this is because the manipulation task that was investigated required significant motions from all four joints. The closed kinematic chain between fingers and object ensured that all joints required inputs of comparable magnitudes and frequency contents.

5 Conclusions and Future Work

In this paper we have proposed a model that explicitly captures interactions between control and communication in computer-controlled LTI systems. For these systems we have computed a family of generalized inverses using an operator-theoretic approach. The generalized inverses were used to solve output tracking problems that arise in systems with limited communication. Possible areas of application for this work include robotic motion control, remotely controlled systems, mobile communications, groups of semi-autonomous vehicles and other areas where communication with the system(s) of interest is limited. Current efforts are focused on exploring models for "closed-loop" controller-plant communication. Our formulation allows for posing joint communication/control optimization problems and for improved tracking performance by choice of an appropriate attention sequence. The price for this, is the apparent difficulty in optimizing with respect to the attention sequence that specifies controller-plant communication. Finding optimal or near-optimal attention sequences for output tracking problems is the subject of ongoing work.

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